

Life inside black holes

V. I. Dokuchaev*

Institute for Nuclear Research of the Russian Academy of Sciences

Abstract

We consider test planet and photon orbits of the third kind inside a black hole, which are stable, periodic and neither come out of the black hole nor terminate at the singularity. Interiors of supermassive black holes may be inhabited by advanced civilizations living on planets with the third-kind orbits. In principle, one can get information from the interiors of black holes by observing their white hole counterparts.

Orbits of the third kind were described in [1, 2, 3, 4, 5] under the assumption of the Kerr-Newman metric validity inside a black hole event horizon. The motion of a test particle (e.g., a planet) with mass μ and electric charge ϵ in the background gravitational field of a Kerr-Newman black hole (BH) with mass M , angular momentum $J = Ma$ and electric charge e is completely defined by three integrals of motion: the total particle energy E , the azimuthal component of the angular momentum L and the Carter constant Q , related to the total angular momentum of the particle. An orbital trajectory of a test planet is governed in the Boyer-Lindquist coordinates (t, r, θ, φ) by the equations of motion [6, 7]:

$$\rho^2 \frac{dr}{d\lambda} = \pm \sqrt{V_r}, \quad \rho^2 \frac{d\theta}{d\lambda} = \pm \sqrt{V_\theta}, \quad (1)$$

$$\rho^2 \frac{d\varphi}{d\lambda} = L \sin^{-2} \theta + a(\Delta^{-1} P - E), \quad (2)$$

$$\rho^2 \frac{dt}{d\lambda} = a(L - aE \sin^2 \theta) + (r^2 + a^2) \Delta^{-1} P, \quad (3)$$

where $\lambda = \tau/\mu$, τ — is the proper time of a particle and

$$V_r = P^2 - \Delta[\mu^2 r^2 + (L - aE)^2 + Q], \quad (4)$$

$$V_\theta = Q - \cos^2 \theta [a^2(\mu^2 - E^2) + L^2 \sin^{-2} \theta], \quad (5)$$

$$P = E(r^2 + a^2) + \epsilon e r - aL, \quad \rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2r + a^2 + e^2. \quad (6)$$

We use the normalized dimensionless variables and parameters: $r \Rightarrow r/M$, $a \Rightarrow a/M$, $e \Rightarrow e/M$, $\epsilon \Rightarrow \epsilon/\mu$, $E \Rightarrow E/\mu$, $L \Rightarrow L/(M\mu)$, $Q \Rightarrow Q/(M^2\mu^2)$. The effective potentials V_r and V_θ in (4) and (5) determine the motion of particles in r - and θ -directions [7]. The Fig. 1 presents examples of third kind nonequatorial orbits of a test planet and a photon, calculated by numerical integration of Eqs. (1) – (3).

For circular orbits of test particles with $r = \text{const}$, Eqs. (4) and (5) provide the conditions:

$$V_r(r) = 0, \quad V_r'(r) \equiv \frac{dV_r}{dr} = 0. \quad (7)$$

The circular orbits would be stable if $V_r'' < 0$, i. e. at the maximum of the effective potential. In the case of a rotating BH (with $a \neq 0$), a particle in an orbit with $r = \text{const}$ may be additionally moving in the latitudinal θ -direction, if $Q \neq 0$. These nonequatorial orbits are called *spherical orbits* [8]. Purely circular orbits correspond to the particular case of spherical orbits with the parameter $Q = 0$. These circular orbits are completely confined in the BH equatorial plane. In general, there are four possible solutions (some of them may be unstable) of Eqs. (7) for

*e-mail: dokuchaev@inr.ac.ru

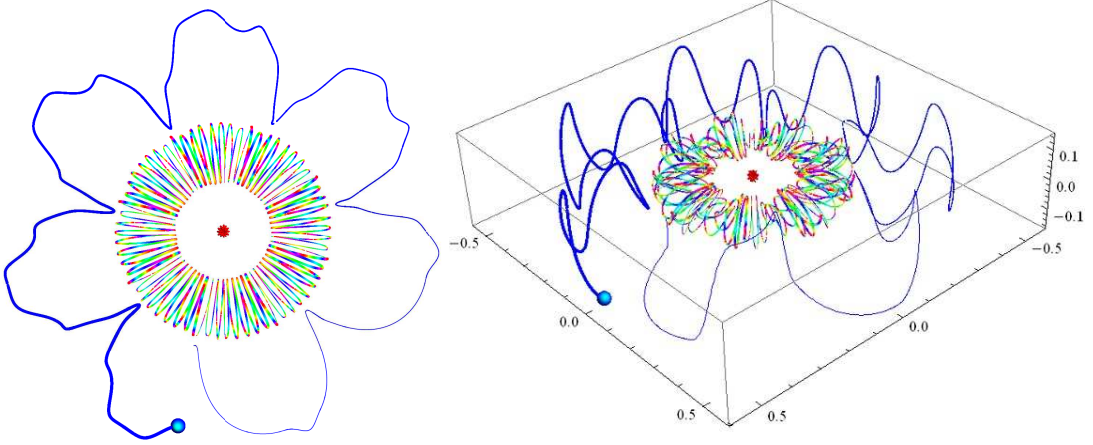


Figure 1: A nonequatorial stable periodic orbit of a planet (the external curve, with $E = 0.568$, $L = 1.13$, $Q = 0.13$) and a photon orbit (the internal curve, with $b = L/E = 1.38$, $q = Q/E^2 = 0.03$) inside a black hole ($a = 0.9982$, $e = 0.05$) in the locally nonrotating frame [7], viewed from the north pole and from the outside.

the azimuthal momentum L_i and the total energy E_i of test particles with a charge ϵ on the spherical orbits with $r = \text{const}$:

$$L_i = -\frac{1}{2} \left(\chi_0 \pm \chi_{1,2} + \frac{1}{2} \frac{\eta_1}{\kappa_1} \right), \quad E_i = \frac{\alpha_1}{\alpha_2} + (\alpha_3 + \alpha_4 L_i + \alpha_5 L_i^2) \frac{L_i}{\alpha_6}, \quad (8)$$

with $i = 1, 2, 3, 4$. The expressions for coefficients χ , η , κ and α in (8) are rather cumbersome:

$$\chi_0 = \sqrt{\left(\frac{1}{4} \frac{\eta_1^2}{\kappa_1} - \frac{2}{3} \xi_1 + \frac{2^{1/3}}{3} \frac{\xi_4}{\xi_6} + \frac{1}{3} \frac{1}{2^{1/3}} \xi_6 \right) \frac{1}{\kappa_1}}, \quad (9)$$

$$\chi_{1,2} = \sqrt{\frac{1}{\kappa_1} \left[\frac{1}{2} \frac{\eta_1^2}{\kappa_1} - \frac{4}{3} \xi_1 - \frac{2^{1/3}}{3} \frac{\xi_4}{\xi_6} - \frac{1}{3} \frac{1}{2^{1/3}} \xi_6 \pm \frac{1}{\chi_0} \left(2\xi_2 + \frac{1}{4} \frac{\eta_1^3}{\kappa_1^2} - \frac{\xi_1 \eta_1}{\kappa_1} \right) \right]}, \quad (10)$$

$$\eta_1 = 8ae\epsilon x^5 \{ 6e^4 + e^2 x(7x-17) + a^2[6e^2 + (x-5)x] + x^2[12 + x(3x-11)] \}, \quad (11)$$

$$\kappa_1 = 4x^6 \{ 4a^2(e^2 - x) + [2e^2 + (x-3)x]^2 \}, \quad (12)$$

$$\begin{aligned} \kappa_2 = & 4 \{ a^4 [Q + (e^2 + Q)x - x^2] + x^3 [Q(2e^2 - 3x) + (e^2 + Q)x^2 - x^3] + a^2 x \{ e^4 + 2e^2 [Q \\ & + (x-2)x] - 2x[Q - (2+Q)x + x^2] \} \}^2 + 4e^2 \epsilon^2 \{ a^8 (Q - x^2)(e^2 + x^2) - x^8 [e^2 \\ & + (x-2)x](Q + x^2) + a^4 x^2 \{ e^6 (\epsilon^2 - 2) - 2x^2 \{ Q[2 + (5-2x)x] + (x-1)x[4 + 3(x-2)x] \} \\ & + e^2 x^2 \{ 8Q - 2[8 + x(5x-12)] + [1 + (x-4)x]\epsilon^2 \} + 2e^4 \{ Q + x[5-3x + (x-1)\epsilon^2] \} \\ & + a^6 \{ Q(x-1-2x^2) - 2x^2 \{ x^2[1 + 2(x-2)x] \} + e^4 [Q + x^2(\epsilon^2 - 3)] + e^2 x \{ Q(4x-2) \\ & + x^2[8 + x(\epsilon^2 - 6)] \} \} - a^2 x^4 \{ 2x^2 \{ Q(3x-5) + x^2[5 + 2(x-3)x] \} \\ & + e^4 (-5Q + x^2(1 + \epsilon^2)) + e^2 x \{ 2Q(7-2x) + x^2[x(6 + \epsilon^2) - 2(4 + \epsilon^2)] \} \} \}, \end{aligned} \quad (13)$$

$$\begin{aligned} \xi_1 = & 4x^2 \{ 4a^6 Q(e^2 - x) + 2x^4 [2e^2 + (x-3)x][2e^2 Q - 3Qx + (e^2 + Q)x^2 - x^3] - e^2 x^6 [e^2 \\ & + (x-2)x]\epsilon^2 + a^4 \{ -2x^2 [x^2(5+x) - Q(x-5)(x-1)] + e^4 [4Q + x^2(13\epsilon^2 - 4)] \\ & + e^2 x \{ 12Q(x-1) + x^2[2(7+x) + (5x-8)\epsilon^2] \} \} a^2 x^2 \{ 4x^2 \{ Q[3 + (x-5)x] \\ & - x[-6 + x(3+x)] \} + e^6 (13\epsilon^2 - 4) + 2e^2 x \{ 2x(x-2)(5+x) \} \} \} \end{aligned}$$

$$+ 2Q(4x-5)+x[11+5(x-3)x\epsilon^2]+2e^4\{4Q+x[11-x+(9x-17)\epsilon^2]\}\}, \quad (14)$$

$$\begin{aligned} \xi_2 = & 8aex\epsilon\left\{a^6\{e^2(2Q-x^2)+x[Q(x-1)+x^2]\}+x^4\{x^2\{x^2(-4+3x)+Q[12+x(3x-11)]\}\right. \\ & + e^4(6Q-x^2(\epsilon^2-1))+e^2x\{Q(7x-17)-x^2[x-1+(x-2)\epsilon^2]\}\}+a^4\{x^2\{x^2(-8+5x) \\ & + Q[4+5(-1+x)x]\}+e^4[2Q+x^2(-4+3\epsilon^2)]+e^2x\{Q(7x-5)+x^2[13-3x+(2x-1)\epsilon^2]\}\} \\ & + a^2x^2\{x^3[16+x(7x-20)+Q(7x-15)]+3e^6(\epsilon^2-1)+e^4\{4Q+x[16-5x+(5x-7)\epsilon^2]\} \\ & \left. + e^2x\{6Q(2x-1)+x\{22x-3x^2-28+[4+3(x-3)x]\epsilon^2\}\}\right\}, \quad (15) \end{aligned}$$

$$\xi_3 = 2\xi_1^3 - 9\xi_2 \xi_1 \eta_1 + 27\kappa_2 \eta_1^2 + 27\xi_2^2 \kappa_1 - 72\kappa_2 \xi_1 \kappa_1, \quad (16)$$

$$\xi_4 = \xi_1^2 - 3\xi_2 \eta_1 + 12\kappa_2 \kappa_1, \quad \xi_5 = \xi_3^2 - 4\xi_4^3, \quad \xi_6 = (\xi_3 + \sqrt{\xi_3^2 - 4\xi_4^3})^{1/3}, \quad (17)$$

$$\begin{aligned} \alpha_1 = & 8ex^2\epsilon\left\{4a^{10}(e^2-x)^2(Q-x^2)+x^{10}(2e^2+(x-3)x)(x(3Q-Qx+x^2)-e^2(2Q+x^2\right. \\ & - x^2\epsilon^2))+a^8\left\{x^3\{x^2[24+(x-27)x]-Q\{8+x[x(6+x)-19]\}\}+e^4x[4Q(5x-3)+x^2(39\right. \\ & - 21x-8\epsilon^2)]+e^2x^2\{3Q[5+(x-14)x]-x^2[55+(x-48)x-4\epsilon^2]\}+4e^6[Q+x^2(\epsilon^2-2)]\} \\ & + a^6x^2\left\{4x^3\{(x-1)x[8+(x-18)x]-Q\{3+x[x(4+x)-16]\}\}+4e^8(\epsilon^2-1) \right. \\ & + e^4x\{Q(44x-56)-4x[17+11(x-3)x]+x[19+(x-24)x]\epsilon^2\}+e^6\{20Q+x[27-23x \\ & + 3(3x-5)\epsilon^2]\}+e^2x^2\{Q[47+x(7x-110)]+x\{76-215x+115x^2-4x^3-[8+(x \\ & - 15)x]\epsilon^2\}\}+a^2x^6\{4x^3[Q(9x-27)+2(3+Q)x^2-(10+Q)x^3+x^4]+4e^6[9Q-2x^2(\epsilon^2 \\ & - 1)]-2e^4x\{-6Q(x-13)+x^2[7+9x+(x-13)\epsilon^2]\}+e^2x^2Q[225-7x(6+x)]+x^2\{-13 \\ & + 53x-4x^2+[x(5+2x)-21]\epsilon^2\}\}+a^4x^4\left\{2x^3\{x\{x[76+3(x-15)x]-24\} \right. \\ & - Q(x-3)[x(13+3x)-8]\}+6q^8(\epsilon^2-1)+q^6\{44Q+x[41-13x+(x-23)\epsilon^2]\} \\ & - q^4x\{Q(152-44x)+x\{104-115x+43x^2+[x(10+x)-29]\epsilon^2\}\}+q^2x^2\{Q[161 \\ & + (x-130)x]+x\{116-245x+123x^2-6x^3+\{x[14+x(3+x)]-12\}\epsilon^2\}\}\left.\right\}, \quad (18) \end{aligned}$$

$$\begin{aligned} \alpha_2 = & 32\left\{\{x^4+a^2[x(2+x)-e^2]\}\{a^2x^2[a^2(x-e^2)+x^2(3x-2e^2)]^2\{Q(x-1) \right. \\ & + x[a^2+e^2+x(2x-3)]\}+a^2[2x^3+a^2(1+x)]\{x^4+a^2[x(2+x)-e^2]\}\{e^2x^2\epsilon^2-[a^2+e^2 \\ & + (x-2)x](Q+x^2)\}-[2x^3+a^2(1+x)]^2\{2a^2(e^2-2x)[a^2+e^2+(x-2)x](Q+x^2) \\ & - e^2x^2[(x-1)x^4+a^4(1+x)+2a^2(e^2-2x+x^3)]\epsilon^2\}\}+[a^2+e^2+(x-2)x][2x^3 \\ & + a^2(1+x)]^3\{e^2x^2\{a^2[e^2-x(2+x)]-x^4\}\epsilon^2-[a^2(e^2-2x)^2(Q+x^2)]\}\left.\right\}, \quad (19) \end{aligned}$$

$$\begin{aligned} \alpha_3 = & -16ax^2\left\{2a^8(e^2-x)^2[Q(1-x)+(e^2-x)x]+x^7[2e^2+(x-3)x]\{(3x-2e^2)[2e^2Q \right. \\ & - 3Qx+(e^2+Q)x^2-x^3]+2e^2x^2[2e^2+x(4x-3)]\epsilon^2\}+a^6x(e^2-x)\left\{2e^6+2e^4x(7x-5) \right. \\ & - x^2\left\{\{Q[15+(x-12)x]\}+x[x(23+x)-8]\}+e^2x\{10Q(x-1)+x[x(37+x)-16]\}\right\} \end{aligned}$$

$$\begin{aligned}
& + 2e^2x\{e^4(x-1)-2e^2(x-1)x+x^3[4+x(3+x)]\}\epsilon^2\} + a^4x^3\left\{10e^8+x^3\{Q[27\right. \\
& + x(5x-28)]-5x[x(15+x)-12]\}+e^6x[-65+29x+10(x-1)\epsilon^2]-e^4x\{4Q(3-5x) \\
& + x\{155-123x-4x^2+[29+x(5x-14)]\epsilon^2\}\}-e^2x^2\{4Q[9+(x-12)x]+x\{\{x[10 \\
& + x(13+10x)]-21\}\epsilon^2+160-169x-9x^2\}\}\} + a^2x^5\{12e^8+x^3\{Q[9+x(7x-36)] \\
& + x[108-x(69+7x)]\}+e^2x^2\{Q[(54-5x)x-21]+2x[x(65+6x)-126] \\
& + x[x^2(16x-13)-27]\epsilon^2\}+e^4x\{4Q(4-5x)+219x+x[4(9+x(5x-6))\epsilon^2 \\
& - 5x(16+x)]\}-4e^6\{Q+x[3(7+\epsilon^2)-4x(1+\epsilon^2)]\}\}\}, \tag{20}
\end{aligned}$$

$$\begin{aligned}
\alpha_4 &= -16ex^7\left\{2x^5[2e^2+(x-3)x]^2+a^4\{4e^4(x-1)-8e^2(x-1)x+x^2\{-3+x[7+x(3+x)]\}\}\right. \\
& \left.- a^2x^2\{12e^4(x-1)+4e^2(x-3)^2x+x^2\{-27+x[9+x(3x-1)]\}\}\right\}\epsilon, \tag{21}
\end{aligned}$$

$$\alpha_5 = 16ax^7[a^2(e^2-x)+(2e^2-3x)x^2]\{4a^2(e^2-x)+[2e^2+(x-3)x]^2\}, \tag{22}$$

$$\begin{aligned}
\alpha_6 &= 32\left\{a^2x^2[a^2(x-e^2)+x^2(3x-2e^2)]^2\{x^4+a^2[x(2+x)-e^2]\}\{Q(x-1)+x[a^2+e^2\right. \\
& + x(2x-3)]\}+a^2[2x^3+a^2(1+x)]\{x^4+a^2[x(2+x)-e^2]\}^2\{e^2x^2\epsilon^2-[a^2+e^2 \\
& + (x-2)x](Q+x^2)\}[2x^3+a^2(1+x)]^2\{x^4+a^2[x(2+x)-e^2]\}\{2a^2(e^2-2x)[a^2+e^2 \\
& + (x-2)x](Q+x^2)-e^2x^2[(x-1)x^4+a^4(1+x)+2a^2(e^2-2x+x^3)]\epsilon^2\}+[a^2+e^2+(x \\
& - 2)x][2x^3+a^2(1+x)]^3\{e^2\epsilon^2x^2\{a^2[e^2-x(2+x)]-x^4\}-[a^2(2x-e^2)^2(Q+x^2)]\}\}. \tag{23}
\end{aligned}$$

The corresponding expressions for L_i and E_i for non charged particles ($\epsilon = 0$) and photons ($\mu = 0$) are much simpler and shorter [9].

Acknowledgements. This research was supported in part by the Russian Foundation for Basic Research grant No. 10-02-00635.

References

- [1] J. Bičák, Z. Stuchlík and V. Balek, *Bull. Astron. Inst. Czechosl.* **40**, 65 (1989).
- [2] V. Balek, J. Bičák and Z. Stuchlík, *Bull. Astron. Inst. Czechosl.* **40**, 133 (1989).
- [3] S. Grunau and V. Kagramanova, *Phys. Rev.* **D83**, 044009 (2011); arXiv:1011.5399 [gr-qc].
- [4] M. Olivares, J. Saavedra, C. Leiva and J. R. Villanueva, *Mod. Phys. Lett. A* **26**, 2923 (2011); arXiv:1101.0748 [gr-qc].
- [5] E. Hackmann, V. Kagramanova, J. Kunz and C. Lämmerzahl, *Phys. Rev.* **D81**, 044020 (2010); arXiv:1009.6117 [gr-qc].
- [6] B. Carter, *Phys. Rev.* **174**, 1550 (1968).
- [7] J. M. Bardeen, W. H. Press and S. A. Teukolsky, *Astrophys. J.* **178**, 347 (1972).
- [8] D. C. Wilkins, *Phys. Rev.* **D5**, 814 (1972).
- [9] V. I. Dokuchaev, *Class. Quantum Grav.* **28**, 235015 (2011); arXiv:1103.6140 [gr-qc].